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Bias-corrected estimation in dynamic panel data models with heteroscedasticity

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Abstract

This study extends earlier results on bias-corrected estimators for the fixed-effects dynamic panel data model. We derive the inconsistency of the LSDV estimator for finite T and N large in case of both time series and cross-section heteroscedasticity and show how to implement the resulting inconsistency expression in bias-correction procedures.

1. Introduction

The inconsistency of the least-squares dummy variable (LSDV) estimator in dynamic panel data models for fixed T has led to the development of a range of new estimators. Various generalized method of moments (GMM) estimators have been proposed and compared (see e.g. Arellano and Bond, 1991; Arellano and Bover, 1995; Ahn and Schmidt, 1995; Blundell and Bond; 1998). In Bun and Carree (2005) a new and simple estimator for dynamic panel data models with or without additional exogenous explanatory variables has been introduced. It is computed as a bias correction to the LSDV estimator (also referred to as within or fixed effects estimator) and is, as such, related to estimators developed by Kiviet (1995), Hansen (2001) and Hahn and Kuersteiner (2002).

Bun and Carree (2005) derive the bias-corrected estimator for finite number of time periods T and large number of cross-section units N under the assumption of homoscedasticity. This paper extends the framework to both time-series and cross-section heteroscedasticity, which are common in applied economic research. The rest of the paper is organized as follows. In section 2 we introduce the model and derive the inconsistency of the LSDV estimator under a variety of assumptions regarding the variance structure of the disturbances. In section 3 we extend the principle of bias-correction developed in Bun and Carree (2005)

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to models with heteroscedastic disturbances. Section 4 contains results from Monte Carlo experiments. Section 5 concludes.

2. Inconsistency of the LSDV estimator

Consider the linear first-order dynamic panel data model with K additional time-varying regressors

$$y_{it} = \gamma y_{i,t-1} + \beta' x_{it} + \eta_i + \varepsilon_{it}, \qquad i = 1, ..., N;\ t = 1, ..., T. \tag{2.1}$$

In this model the dependent variable y_{it} is determined by the one-period lagged value of the dependent variable $y_{i,t-1}$,a $K \times 1$ vector of explanatory variables x_{it}, an individual specific effect η_i and a mean zero error term ε_{it} with variance σ^2_{it}. Stacking the observations over time we get

$$y_i = \gamma y_{i,-1} + X_i\beta + \eta_i \iota_T + \varepsilon_i, \qquad i = 1, ..., N, \tag{2.2}$$

where $y_i = (y_{i1}, ..., y_{iT})'$, $y_{i,-1} = (y_{i0}, ..., y_{i,T-1})'$, $X_i = (x_{i1}, ..., x_{iT})'$, $\varepsilon_i = (\varepsilon_{i1}, ..., \varepsilon_{iT})'$ and $\iota_T = (1, ..., 1)'$ is a $T \times 1$ vector of ones. We assume that conditional on the observables y_{i0} and X_i and the unobservables η_i the disturbance term ε_i is independently distributed across individuals with mean zero and covariance matrix Σ_i. We allow for both time-series and cross-section heteroscedasticity in the following way:

$$\Sigma_i = diag(\sigma^2_{it}) \text{ with } \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \Sigma_i = \Sigma_T = diag(\sigma^2_t). \tag{2.3}$$

Hence, regarding cross-sectional heteroscedasticity we follow Phillips and Sul (2004, Assumption A1), but extend their assumption to allow for time series heteroscedasticity too. Phillips and Sul (2004) show that the particular form of the inconsistency of the LSDV estimator in case of cross-sectional heteroscedasticity does not change. Below we show that in case of time series heteroscedasticity it does, however, which is relevant when developing bias-corrected procedures as will be shown in the next section.

Stacking the observations once again across individuals one gets

$$y = \gamma y_{-1} + X\beta + (I_N \otimes \iota_T)\eta + \varepsilon, \tag{2.4}$$

where y and y_{-1} are $NT \times 1$ vectors of stacked observations, X is a $NT \times K$ matrix, $\eta = (\eta_1, ..., \eta_N)'$ and ε is a $NT \times 1$ vector of disturbances. Define $A = I_N \otimes A_T$ with $A_T = I_T - \frac{1}{T}\iota_T\iota_T'$ as the within transformation which eliminates the individual effects. The LSDV coefficient estimator of γ and β in model (2.4) is equal to ordinary least squares on the transformed model

$$\tilde{y} = \gamma\tilde{y}_{-1} + \tilde{X}\beta + \tilde{\varepsilon}, \tag{2.5}$$

where $\tilde{y} = Ay$, $\tilde{y}_{-1} = Ay_{-1}$, $\tilde{X} = AX$ and $\tilde{\varepsilon} = A\varepsilon$.

The LSDV estimators are biased and inconsistent for T finite and N large because $\tilde{y}_{-1}$ and $\tilde{\varepsilon}$ are correlated. Results on the extent of the inconsistency have been derived by Nickell (1981) and Kiviet (1995) assuming *iid* disturbances ε_{it}. Using partitioned regression techniques the LSDV estimation errors of γ and β in (2.5) can be expressed as (see also Nickell, 1981)

$$\begin{aligned}\hat{\gamma} - \gamma &= (\tilde{y}'_{-1}M\tilde{y}_{-1})^{-1}\tilde{y}'_{-1}M\tilde{\varepsilon},\\ \hat{\beta} - \beta &= -(\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y}_{-1}(\hat{\gamma} - \gamma) + (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{\varepsilon},\end{aligned}$$

where $M = I - \tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}'$. Hence, the inconsistency reads

$$\left.\begin{array}{r}\text{plim}_{N\to\infty}(\hat{\gamma}-\gamma) = \left(\text{plim}_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}'M\tilde{y}_{-1}\right)^{-1}\text{plim}_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}'M\tilde{\varepsilon} \\ \text{plim}_{N\to\infty}(\hat{\beta}-\beta) = -\,\text{plim}_{N\to\infty}(\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y}_{-1}\,\text{plim}_{N\to\infty}(\hat{\gamma}-\gamma)\end{array}\right\}, \tag{2.6}$$

from which it is seen that the inconsistency critically depends on $\text{plim}_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}'M\tilde{\varepsilon}$. Because of the assumed strict exogeneity of X this term can be written as $\text{plim}_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}'M\tilde{\varepsilon} = \text{plim}_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}'\tilde{\varepsilon}$.

For further assessment of the inconsistency of the LSDV estimator (2.11) we have to evaluate

$$\text{plim}_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}'\tilde{\varepsilon} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}E\left[\tilde{y}_{i,-1}'\tilde{\varepsilon}_i\right],$$

hence we need a decomposition of the transformed regressor $\tilde{y}_{-1}$ into two parts, i.e. correlated with $\tilde{\varepsilon}$ or not. From (2.2) it is seen that

$$y_i = \gamma\left(L_T y_i + y_{i0}e_T\right) + X_i\beta + \eta_i\iota_T + \varepsilon_i, \tag{2.7}$$

where we introduced a $T\times T$ matrix L_T with ones on the first lower subdiagonal and zeros elsewhere and where e_T is the $T\times 1$ unit vector with its first element equal to one. Defining $\Gamma_T = (I_T - \gamma L_T)^{-1}$ we can write

$$y_i = \Gamma_T\left(\gamma y_{i0}e_T + X_i\beta + \eta_i\iota_T + \varepsilon_i\right). \tag{2.8}$$

Furthermore, we have

$$\begin{aligned}\tilde{y}_{i,-1} &= A_T(L_T y_i + y_{i0}e_T)\\ &= A_T(L_T\Gamma_T\left(\gamma y_{i0}e_T + X_i\beta + \eta_i\iota_T + \varepsilon_i\right) + y_{i0}e_T)\\ &= \Pi_T\varepsilon_i + v_i\end{aligned} \tag{2.9}$$

where $\Pi_T = A_T L_T\Gamma_T$ and $v_i = y_{i0}(\gamma\Pi_T + A_T)e_T + \Pi_T X_i\beta + \eta_i\Pi_T\iota_T$. Note that under the assumptions made ε_i and v_i are uncorrelated and that the elements of Π_T depend on γ only.

Using the decomposition (2.9) we have

$$E\left[\tilde{y}_{i,-1}'\tilde{\varepsilon}_i\right] = tr\left(\Pi_T\Sigma_i\right). \tag{2.10}$$

Further evaluation of the expectation in (2.10) requires an explicit assumption about the variance structure of ε_i. Using (2.3) we have

$$\begin{aligned}\text{plim}_{N\to\infty}\frac{1}{N}\tilde{y}_{-1}'\tilde{\varepsilon} &= \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}tr(\Pi_T\Sigma_i) = tr\left(\Pi_T\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\Sigma_i\right)\\ &= tr(\Pi_T\Sigma_T),\end{aligned}$$

hence, as shown by Phillips and Sul (2004) already, the presence of cross-sectional heteroscedasticity does not have consequences for the particular form of the inconsistency. In case of time series heteroscedasticity, i.e. $\Sigma_T = diag(\sigma_t^2)$, we derive

$$tr(\Pi_T\Sigma_T) = -\sum_{j=0}^{T-2}\sigma_{T-1-j}^2\sum_{s=0}^{j}\gamma^s.$$

Note that assuming homoscedasticity, i.e. $\Sigma_T = \sigma^2 I_T$, it is easily seen that

$$\text{plim}_{N\to\infty} \frac{1}{N}\tilde{y}_{-1}'\tilde{\varepsilon} = \sigma^2 tr(\Pi_T) = -\sigma^2 \left(\frac{1}{1-\gamma} - \frac{1-\gamma^T}{T\,(1-\gamma)^2} \right),$$

which has been derived before in Nickell (1981) and Kiviet (1995).

Summarizing the results, while cross-section heteroscedasticity does not alter the specific form of the inconsistency, time-series heteroscedasticity does. Note that, although precise form of the inconsistency changes, also in case of heteroscedasticity it is $O(T^{-1})$. Defining $\Sigma_{xy_{-1}} = \text{plim}_{N\to\infty} \frac{1}{N}\tilde{X}'\tilde{y}_{-1}$, $\Sigma_{xx} = \text{plim}_{N\to\infty} \frac{1}{N}\tilde{X}'\tilde{X}$, $\sigma^2_{y_{-1}} = \text{plim}_{N\to\infty} \frac{1}{N}\tilde{y}_{-1}'\tilde{y}_{-1}$ the inconsistency of the LSDV estimator (2.6) can be expressed as

$$\left.\begin{aligned} \text{plim}_{N\to\infty}(\hat{\gamma}-\gamma) &= tr(\Pi_T\Sigma_T)/\sigma^2_{y_{-1}|X} \\ \text{plim}_{N\to\infty}(\hat{\beta}-\beta) &= -\zeta\, \text{plim}_{N\to\infty}(\hat{\gamma}-\gamma) \end{aligned}\right\}, \tag{2.11}$$

where we introduced $\sigma^2_{y_{-1}|X} = (1-\rho^2_{Xy_{-1}})\sigma^2_{y_{-1}}$ as the conditional variance of $\tilde{y}_{-1}$, $\rho^2_{Xy_{-1}} = \Sigma'_{xy_{-1}}\Sigma^{-1}_{xx}\Sigma_{xy_{-1}}/\sigma^2_{y_{-1}}$ as the (asymptotic) squared multiple correlation coefficient of the regression of $\tilde{y}_{-1}$ on $\tilde{X}$ and $\zeta = \Sigma^{-1}_{xx}\Sigma_{xy_{-1}}$ as the corresponding vector of regression coefficients.

3. Bias correction in case of heteroscedasticity

We now turn to bias-corrected estimation of γ and β. We will use (2.11) to develop both linear and nonlinear bias corrections. The former closely corresponds with the bias correction proposed by Kiviet (1995), while the latter is a straightforward application of the method proposed in Bun and Carree (2005). Both studies assumed homoscedasticity, but we will extend their methods to models with heteroscedastic disturbances.

First, the additive bias corrected estimator is constructed as the original LSDV estimator minus an estimate of the inconsistency (2.11). For that we need preliminary consistent estimates of $\sigma^2_{y_{-1}|X}$, ζ, Π_T and Σ_T. The first two quantities can be estimated consistently using their sample analogs $\hat{\sigma}^2_{y_{-1}|X}$ and $\hat{\zeta}$. Regarding Π_T and Σ_T GMM coefficient estimators (labelled $\hat{\gamma}_{gmm}$ and $\hat{\beta}_{gmm}$) are used for providing first step consistent estimates. Π_T is depending on γ only, hence we use $\hat{\gamma}_{gmm}$ to provide a consistent estimate $\hat{\Pi}_{T,gmm}$. Σ_T can be estimated consistently from the GMM residuals by

$$\hat{\Sigma}_{T,gmm} = diag\left(\hat{\sigma}^2_{t,gmm}\right), \tag{3.1}$$

$$\hat{\sigma}^2_{t,gmm} = \frac{(\tilde{y}_t - \hat{\gamma}_{gmm}\tilde{y}_{t-1} - \tilde{X}_t\hat{\beta}_{gmm})'(\tilde{y}_t - \hat{\gamma}_{gmm}\tilde{y}_{t-1} - \tilde{X}_t\hat{\beta}_{gmm})}{N(T-1)/T},$$

where $\tilde{y}_t = (\tilde{y}_{1t}, ..., \tilde{y}_{Nt})'$, $\tilde{y}_{t-1} = (\tilde{y}_{1,t-1}, ..., \tilde{y}_{N,t-1})'$, $\tilde{X}_t = (\tilde{x}_{1t}, ..., \tilde{x}_{Nt})'$.Using (2.11) the additive bias-corrected estimator (*abc*) for γ and β is

$$\left.\begin{aligned} \hat{\gamma}_{abc} &= \hat{\gamma}_{lsdv} - \frac{tr\left(\hat{\Pi}_{T,gmm}\hat{\Sigma}_{T,gmm}\right)}{\hat{\sigma}^2_{y_{-1}|X}} \\ \hat{\beta}_{abc} &= \hat{\beta}_{lsdv} + \hat{\zeta}\frac{tr(\hat{\Pi}_{T,gmm}\hat{\Sigma}_{T,gmm})}{\hat{\sigma}^2_{y_{-1}|X}} \end{aligned}\right\}. \tag{3.2}$$

Note that we use a slightly different version of Kiviet's (1995) estimator, i.e. there is bias correction of the first-order term (2.11) only and higher order bias terms are neglected. Bun and Kiviet (2002), however, show that this first-order term is responsible for the majority of the finite sample bias in the LSDV estimator.

Second, regarding the nonlinear bias correction we first assume the variance structure Σ_T to be given as $\sigma^2_{y_{-1}|X}$ and ζ. Hence, the only unknown quantities in (2.11) are γ and β. Using the first expression of (2.11) the bias-corrected estimator for γ is that γ which solves

$$\hat{\gamma} = \gamma - \frac{tr(\Pi_T \Sigma_T)}{\sigma^2_{y_{-1}|X}}, \tag{3.3}$$

The resulting estimator can then be inserted into the second expression in (2.11) to find the bias-corrected estimator for β. In general $\sigma^2_{y_{-1}|X}$, ζ and Σ_T are unknown and have to be estimated too. As before the first two quantities can be estimated consistently using their sample analogs. Regarding Σ_T we use the consistent, although infeasible, estimator

$$\hat{\Sigma}_T = diag\left(\hat{\sigma}^2_t\right), \tag{3.4}$$

$$\hat{\sigma}^2_t = \frac{(\tilde{y}_t - \gamma\tilde{y}_{t-1} - \tilde{X}_t\beta)'(\tilde{y}_t - \gamma\tilde{y}_{t-1} - \tilde{X}_t\beta)}{N\left(T-1\right)/T}.$$

We then solve the following system of $K+1$ equations simultaneously for γ and β:

$$\left.\begin{aligned} \hat{\gamma}_{lsdv} &= \gamma - tr(\Pi_T\hat{\Sigma}_T)\hat{\sigma}^2_{y_{-1}|X} \\ \hat{\beta}_{lsdv} &= \beta + \hat{\zeta} tr(\Pi_T\hat{\Sigma}_T)\hat{\sigma}^2_{y_{-1}|X} \end{aligned}\right\}. \tag{3.5}$$

We will label the solutions $\hat{\gamma}_{nbc}$ and $\hat{\beta}_{nbc}$. Note that in case of homoscedasticity the resulting procedure is equal to one proposed in Bun and Carree (2005).

4. Monte Carlo experiments

In this section we compare the performance of the additive and nonlinear bias-corrected estimators (labelled *abc* and *nbc* respectively) with some alternative estimators. We compare it with (i) the LSDV-estimator (*lsdv*), (ii) the GMM-estimator (*gmm*) by Arellano and Bond (1991). Assuming strict exogeneity of x_{it} we have $T(T-1)/2+T(T-1)$ moment conditions for *gmm*, i.e. $E\left[y_{i,t-s}\Delta\varepsilon_{it}\right] = 0$ $(t = 2, ..., T; s = 2, ..., t)$ and $E\left[x_{is}\Delta\varepsilon_{it}\right] = 0$ $(t = 2, ..., T; s = 1, ..., T)$. Additional moment conditions due to imposing homoscedasticity are not exploited as Ahn and Schmidt (1995) note that efficiency gains are small. Under the assumptions made in section 2 the GMM-estimator is consistent for finite T and N large, hence it is a reasonable benchmark for evaluating the bias-corrected estimators.

Data for y have been generated according to equation (2.1) with $\eta_i \sim \mathcal{IIN}[0, \sigma^2_\eta]$ and $\varepsilon_{it} \sim \mathcal{IIN}[0, \sigma^2_{it}]$. The generating equation for the explanatory variable x is

$$x_{it} = \rho x_{i,t-1} + \xi_{it}, \qquad i = 1, ..., N;\ t = 1, ..., T, \tag{4.1}$$

where $\xi_{it} \sim \mathcal{IIN}[0, \sigma^2_\xi]$. We choose $\gamma = 0.8$, $\beta = 1$, $\rho = 0.8$ and $\sigma_\eta = \sigma_\xi = 1$. We assume that the panel data set has 600 observations and conduct experiments for several combinations of T and N for which $NT = 600$.

Regarding the disturbance variance structure σ_{it}^2 we use two research designs, i.e. (1) cross-sectional heteroscedasticity and (2) time series heteroscedasticity. For design 1 we specify $\sigma_{it}^2 = \sigma_i^2 \sim \chi^2(1)$ and for design 2 we have $\sigma_{it}^2 = \sigma_t^2 = 0.95 - 0.05T + 0.1t$. In these specifications it is ensured that $\frac{1}{N}\sum_{i=1}^{N} \sigma_{it}^2 \approx 1$ and $\frac{1}{T}\sum_{t=1}^{T} \sigma_{it}^2 = 1$.

Simulation results are presented in Tables 1 and 2. Regarding coefficient estimators we present in these tables the bias in estimating γ and β together with the root mean squared error (RMSE). In calculating the RMSE of coefficient estimators we use the variance as estimated from the Monte Carlo as a measure of true variance. For each experiment we performed 10,000 Monte Carlo replications.

We observe the following patterns in the simulation results for the coefficient estimators. First, bias in estimating the autoregressive parameter γ is negative for *lsdv* and *gmm*. Second, regarding (bias-corrected) LSDV bias in estimating both γ and β decreases for larger T (and smaller N), but not for *gmm*. This is to be expected as *gmm* should perform well especially for T small and N large. Third, in estimating both γ and β both bias-corrected estimators are virtually unbiased. Finally, based on a mean squared error criterion bias corrected estimators are efficient compared with *gmm* coefficient estimators.

5. Concluding remarks

This study has developed bias-corrected estimation techniques for the LSDV estimator in the dynamic panel data model with heteroscedastic disturbances. The inconsistency of the LSDV estimator for finite T and N large is derived under both time series and cross-section heteroscedasticity. The resulting expressions are used in extending existing additive and nonlinear bias correction procedures. The resulting bias-corrected estimators are consistent for finite T and N large. We provided some simulation results allowing for either cross-section or time-series heteroscedasticity. From the simulation results it is seen that the proposed bias corrected estimators behave satisfactorily in finite samples. Simulation results on various designs show that based on a root mean squared criterion bias-corrected LSDV estimators perform well against GMM estimators using the same assumptions.

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Table 1: Cross-section heteroscedasticity, $\gamma = \rho = 0.8$ and $\beta = 1$

(N,T)	$(300,2)$	$(200,3)$	$(150,4)$	$(100,6)$	$(60,10)$	$(40,15)$
			bias γ			
lsdv	-0.363	-0.214	-0.142	-0.079	-0.038	-0.021
ac	0.003	-0.002	-0.002	-0.002	-0.002	-0.002
bc	0.007	0.001	0.001	0.000	-0.001	-0.000
gmm	-0.003	-0.010	-0.012	-0.014	-0.017	-0.017
			RMSE γ			
lsdv	0.369	0.218	0.147	0.083	0.042	0.026
ac	0.075	0.047	0.035	0.024	0.017	0.014
bc	0.091	0.051	0.038	0.025	0.017	0.014
gmm	0.071	0.046	0.037	0.028	0.025	0.023
			bias β			
lsdv	-0.101	-0.031	-0.004	0.015	0.021	0.019
ac	0.001	0.000	0.000	0.000	0.001	0.002
bc	0.002	0.001	0.000	-0.000	0.000	0.001
gmm	-0.001	-0.001	-0.000	0.003	0.009	0.015
			RMSE β			
lsdv	0.124	0.066	0.051	0.047	0.043	0.040
ac	0.081	0.061	0.051	0.044	0.038	0.035
bc	0.083	0.061	0.051	0.044	0.038	0.035
gmm	0.081	0.061	0.051	0.044	0.039	0.038

Note: We assume $\sigma_i^2 \sim \chi^2(1)$, $\sigma_\eta^2 = \sigma_\xi^2 = 1$

Table 2: Time-series heteroscedasticity, $\gamma = \rho = 0.8$ and $\beta = 1$

(N, T)	$(300, 2)$	$(200, 3)$	$(150, 4)$	$(100, 6)$	$(60, 10)$	$(40, 15)$
			bias γ			
lsdv	-0.353	-0.203	-0.133	-0.072	-0.033	-0.018
ac	0.021	0.005	0.003	0.000	-0.001	-0.001
bc	0.035	0.010	0.006	0.002	0.000	-0.000
gmm	-0.002	-0.008	-0.009	-0.010	-0.013	-0.014
			RMSE γ			
lsdv	0.356	0.206	0.136	0.075	0.036	0.022
ac	0.072	0.043	0.033	0.023	0.016	0.013
bc	0.084	0.047	0.034	0.023	0.016	0.013
gmm	0.072	0.046	0.036	0.026	0.021	0.020
			bias β			
lsdv	-0.098	-0.029	-0.003	0.013	0.018	0.015
ac	0.006	0.002	0.001	-0.001	0.001	0.001
bc	0.010	0.003	0.001	-0.001	0.000	-0.000
gmm	-0.000	0.001	0.001	0.001	0.007	0.012
			RMSE β			
lsdv	0.121	0.066	0.052	0.046	0.042	0.038
ac	0.082	0.061	0.052	0.044	0.038	0.034
bc	0.084	0.061	0.052	0.044	0.038	0.034
gmm	0.081	0.060	0.052	0.044	0.039	0.037

Note: We assume $\sigma_t^2 = 0.95 - 0.05T + 0.1t$, $\sigma_\eta^2 = \sigma_\xi^2 = 1$