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Abstract

— This note contains some corrections and amplifications that were deduced from the implementation and testing of the method.

Keywords: Matrix differentiation; Vector autoregressive moving average model; Fisher information matrix.

1. Introduction

During implementation of the method described in Klein *et al.* [1], some minor mistakes became apparent and also a severe failure concerning the initialization stage in Section 5, Remark 4.

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2. Some minor corrections

(1) In (48), last term, replace I_n by I_m

(2) In (54), line 3, replace F^T by $\overline{F}_t^T$

(3) Also in (54), last term, replace $M_{mn,n}$ by $M_{mn,m}$

(4) In (58), first line, replace M_{n,n^2} by $M_{n^2,n}$

(5) On page 229, ten occurences of vecF^θ should be replaced by F^θ but most of these relations are useless, as seen in the next Section.

3. Modified initialization stage

Indeed, the initialization stage described in Section 5, Remark 4, is failed. It worked perfectly for the simple example (submitted in the first version of the paper but omitted at referee's request), a vector moving average (VMA) of order 1, i.e. the case where $p = 0$ and $s = 1$. But it proved to be wrong when comparing the results for more general univariate ARMA processes with the program (Pmeklex, due to Zahaf) built on the basis of [3]. The whole page 229, except the last sentence, should be replaced by the following:

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Note that $\mathbb{E}\{w_1 \otimes w_1\} = \text{vec}Q$. Initial covariances involving only the process $\widetilde{x}_t$, like $\mathbb{E}\{\widetilde{x}_1 \otimes \widetilde{x}_1\}$ or $\mathbb{E}\left\{\left(\widetilde{x}_1^\theta\right)^T \otimes \widetilde{x}_1\right\}$ or $\mathbb{E}\{\widetilde{x}_1^\theta \otimes \widetilde{x}_1^\theta\}$, make use of the initial values discussed in Remarks 1 and 2. For example $\mathbb{E}\{\widetilde{x}_1 \otimes \widetilde{x}_1\} = P_{1|0}$. Note that the process x_t is stationary and doesn't depend on the observations made at times $t = 1, ..., N$.

In order to have a stable solution for all the equations involving the process x_t, we can run the recurrence equations of Section 4 for $t = 1, ..., N_0$, where N_0 is large enough, watching for convergence. This needs initial values which can be any non-zero matrices of appropriate sizes. When we have the stable solutions, we use them as the initial values of the recurrence equations of Section 4 but for $t = 1, ..., N$, where N is the number of observations.

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Indeed, it can be seen that the original initialization $x_1 = Fw_0$ is not compatible with $\mathbb{E}\left\{\tilde{z}_1\tilde{z}_1^T\right\} = B_1$ and is therefore wrong. For example, for a univariate AR(1) process (i.e. the case where $m = 1, p = 1, s = 0$) described by the equation $z_t = \theta_1 z_{t-1} + w_t$ with $Q = 1$, the state vector is $x_t = \theta_1 z_{t-1}$ and its variance is $\theta_1^2/(1-\theta_1^2)$. When we replace Φ and F by θ_1 in (65) we obtain effectively $\theta_1^2/(1-\theta_1^2)$ as a stable solution of that equation. However, following the wrong original approach with the stated initial value $\mathbb{E}\left[x_1 \otimes x_1\right] = (F \otimes F)\ \mathrm{vec}Q = \theta_1^2$, we have of course a time-dependent solution which is not appropriate. This illustrates the mistake. Another solution is possible but it would involve $P_{1|0}$ which is not fully computed here and similar matrices for expectations involving derivatives $\widetilde{x}_1^{\theta}$.

4. Conclusion

A related paper [2] contains a discussion of the implementation in the MATLAB environment as well as numerical aspects related to it. Note that computing the derivatives of the autocovariances is the subject of an other paper [4] where some algebraic aspects related to the problem are mentioned.

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